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ROCKY MOUNTAIN FOREST AND RANGE EXPERIMENT STATION

Height-Diameter Curves for Tree Species Subject to Stagnation

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Relationships between tree heights and diameters in even-aged stands of ponderosa and lodgepole pines can be expressed by the equation H = a + b log D.

Forest inventory instructions usually specify that only a sample of tree heights will be measured. Unmeasured heights are estimated from a graphical or mathematical relationship between height and diameter. Mathematical solutions are especially useful where a specific height-diameter relationship applies over a wide area. Standardized instructions can be included in computer programs, or used to guide least squares solutions on desk calculators. Workers who use graphical methods often find useful relationships represented on commercial graph paper. Linear and log-function graph papers make it easy to plot data for graphical solutions.

Comparison of several height-diameter relationships showed the following equation to be the most useful for even-aged stands of ponderosa (Pinus ponderosa Laws.) and lodgepole (P. contorta Dougl.) pines:

$$H = a + b \log D$$

where

H = total height,
D = diameter at breast height,
a and b = regression constants.

Form of the relationship is not changed by variation in tree size or stand density. The model applies to stands so dense that height growth is reduced, as well as to those kept vigorous by repeated thinning. Relatively simple solutions of the height-diameter relationship are therefore possible, either by machine or on semilogarithmic paper.

Relationships recommended in American and European publications are tested to determine those most useful where stagnation of

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growth is possible. Equations tested included the following:

$$H = a + b \log D$$

$$H = a + b/D^2$$

$$H = a + b\ D^2$$

$$H = a + b_1 D - b_2 D^2$$

plus multiple regressions involving two or more transformations of D.

Data from 91 ponderosa pine plots in the Black Hills of South Dakota and Wyoming were analyzed by least squares. Regressions with the lowest standard errors and best fit between measured and computed heights were identified for each plot. The ponderosa pine plots sampled wide ranges of stand characteristics:

Basal area--57 to 261 square feet.
Average stand diameter--2.0 to 17.5 inches.
Average stand height--13 to 85 feet.
Stand age--25 to 161 years.
Site index--36 to 76 feet (base 100 years).

Best solutions for the 91 ponderosa pine plots were distributed as follows:

Transform of diameter measure	Number of plots where transform was: Best	Tied for best
log D	33	45
$D - D^2$	5	23
$1/D^2$	2	20
D^2	3	17

Log D was the best expression of diameter tested for 33 of the 91 sets of data. It was equal to or better than other forms of expression in 78 cases.

The frequently used relationship $H = a + b_1 D - b_2 D^2$ did not rank high. D and D^2 were both significant independent variables for only 37 of the 91 sets of data. Other multiple regressions tested did not produce valid relationships.

The 13 plots where log D was not as good as other transforms of diameter had no measured stand characteristics in common. Characteristics examined were: Average stand diameter, average height, dominant height, age, stand density, site index, and thinning history. In no case did a solution based on log D give results that were unacceptable for the usual purposes for which estimated heights are used. Standard errors were 0.7 to 3.1 feet.

Data from 47 lodgepole pine plots in Colorado and Wyoming were used to verify results of the ponderosa pine analysis. Stands varied much less in average diameter and other characteristics than did the ponderosa pine stands, but both dense and thinned stands were included. Height-diameter relationships of 45 of the 47 plots were closely represented by $H = a + b \log D$. On two plots, computed heights of the largest and smallest diameter classes differed from measured heights by 3 to 4 feet. There were only one or two trees in each of these classes, both on the two plots and in the sample.